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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The research on this grant explored homotopy continuation methods as a way of solving systems of nonlinear equations. One paper published under this grant describes regularity properties when the homotopy is analytic. Another describes homotopies for which the functions are piecewise analytic.

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FINAL REPORT ON GRANT DAAG-29-80-C-0040

The senior personnel supported on this grant were J. A. Yorke (P.I.) and T. Y. Li. Papers [1] and [2] were written under this grant. In addition S. Pelikan and I. Schwartz were graduate students who were supported in part under this grant. The main results in Schwartz's thesis are described in [3]. Hence, papers [1, 2, 3] have been accepted for publication and were supported by this grant.

The objective of the work done under this grant has been to explore homotopy continuation methods. Chow, Mallet-Paret, and Yorke gave a general approach in [4] for using the homotopy method for solving for zeros of smooth maps. Their approach will generally give a solution for problems in which the existence of a solution can be demonstrated by topological degree arguments. The basic idea in their approach is simple and can be described briefly as follows. Let $f : R^n \to R^n$ be a C^1 map for which we want to find a zero. Assume we are given a "trivial" mapping $g_a : R^n \to R^n$ whose zeros are known a priori, depending on parameter a. Define the homotopy

$$\phi_a(\lambda,x) = (1 - \lambda)g_a(x) + \lambda f(x), \quad 0 \le \lambda \le 1 . \tag{1}$$

They use the parametrized Sard Theorem to obtain a guarantee that for almost every a, $\phi_a^{-1}(0)$ consists of one or more disjoint simple smooth curves in λ,x space. The curve containing $(0,a)$ can sometimes be shown to lead from a zero of g_a at $\lambda = 0$ to a zero of f at $\lambda = 1$. Their idea is that one should start with a <u>large</u> class of trivial maps g_a. Generally the parameter space should be at least n dimensional. Next choose a parameter value a at random (using a pseudo random number generator for example). Next homotop the zeroes of g_a to the zeroes of the nontrivial map. With probability one, the curve Γ_a of zeroes of (1) starting from the zero of the trivial map will be a smooth path without bifurcation.

In [1] two main features of the curves Γ_a described above are studied in the case where $\phi_a(\lambda,x)$ is real analytic with respect to both λ and x. First if $f(x_0) = 0$ and the curve Γ_a has $(1,x_0)$, as a limit point, we show that there is a neighborhood N of $(1,x_0)$ in (λ,x) space for which

$$N \cap \phi_a^{-1}(0) \cap \{(\lambda,x) : 0 \leq \lambda < 1\}$$

is the union of finitely many curves, each of which has an analytic parametrization. That is, it consists of finitely many $y_i(t)$, $0 \leq t < t_i$, with $y_i(0) = (1,x_0)$ and $y_i(t) = (1 - t, x_i(t))$ where each $x_i(t)$ is given by a convergent fractional power series in t that is a convergent power series in $t^{1/N}$ for some integer N. Secondly, let $G = \{a{:}0$ is a regular value of $\phi_a\}$; notice that if $a \in G$, then $\phi_a^{-1}(0)$ consists of smooth curves. We prove G is open and dense in the space of all a. This property is of particular importance in computer implementation.

A number of papers develop a class of continuation methods for solving nonlinear systems of equations which have the feature that, under broad topological assumptions which guarantee the existence of solutions of the system, the methods are guaranteed with probability one to generate a curve which approaches arbitrarily close to a solution of the system. In these papers, it is assumed that the nonlinear system is defined by smooth functions. Piecewise linear techniques are similarly used.

The purpose of this paper is to develop path following methods for a class of problems including both piecewise linear and smooth systems of equations. We formulate the method for "piecewise smooth functions" on a "piecewise smooth domain," and we give similar guaranteed convergence results.

As an illustration of the kinds of problems we want to be able to handle, we let B be the ball in R^n and let $f: B \rightarrow B$ be piecewise smooth in the sense defined in the next section. (In particular we assume f is continuous.) Following the homotopy approach formally, we choose $z \in B$ and write the homotopy

$$F_z(x,t) = (1-t)z + tf(x) - x$$

where $t \in [0,1]$. The zeroes of $F_z(1,x)$ are the fixed points of f while z is the unique zero of $F_z(0,x)$. When f is smooth (C^2), it is shown in [4] that for almost every $z \in B$ a smooth path in B x [0,1] leads from (0,z) to at least one zero at t = 1. The objective of this paper is to develop a corresponding theory which permits f to be piecewise smooth and to show there is a piecewise smooth path of zeroes of F_z that leads to a fixed point (or possibly to a larger set of fixed points) of f. The facts about the paths for F_z follow from the general theory we develop here, and we develop only enough theory for us to handle applications. We give applications in [2] to show how the piecewise smooth formulation can be used, and these are discussed in detail. First we consider the nonlinear complementarity problem. We put it in our context and prove an existence result. The continuation method we develop is a nonlinear form of Lemke's algorithm. Second we consider nonlinear constrained optimization.

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